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DOUBLY BALANCED TERNARY DESIGNS AND THEIR BALANCED ARRAYS WITH APPLICATIONS TO INTERCROPPING EXPERIMENTS

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ABSTRACT

This paper is concerned with the recursive construction of a series of doubly balanced ternary(DBT) designs through balanced ternary designs using a set of balanced incomplete block (BIB) designs giving the relationship among the parameters. Some illustrative examples have been added . The applications of the designs have been discussed to the intercropping experiments.

Keywords: Balanced incomplete block design (BIBD), Self complimentary BIB design, Balanced Ternary design (BTD),Doubly balanced ternary design(DBTD), Incidence matrix, Intercropping experiment.

1. Introduction

Balanced *n-ary* designs were introduced by Tocher (1952) as a generalization of balanced incomplete block design. In this design, the incidence matrix can take any value of *n* out of possible values, often 0,1,2….,(n-1). If n = 3, we get a ternary block design. Billington (1984,1989) have extensively given results on balanced ternary designs. These designs may not exist for all parametric combinations or even if exist may require a large number of replications. In the present paper, we provide a new method of recursive construction of DBTdesigns through balanced ternary designs using a set of balanced incomplete block designs.

2. Definition and Notations

2.1 Balanced Ternary Design

A balanced *n-ary* design with parameters V,B,R,K & $\wedge$ and incidence matrix N=((n_{ij})) is an arrangement of V treatments in B blocks, each of cardinality K(K $\leq$ V) such that (i) the i^{th} treatment appears n_{ij} times in the j^{th} block where n_{ij} can take any of the values 0,1,2,...n-1. (ii) each treatment occurs R times, and (iii) $\sum_{j=1}^{j=B} n\, ijn\, i'\, j = \wedge 2$ for all i $\neq$ i' = 1, 2, ...V. It is to be noted that $\sum_{j=1}^{j=B} n_{ij} =$ R for all i and $\sum_{j=1}^{j=V} n_{ij} = K$ for all *i*. For n = 2 (binary) the design is called a BIB design with the usual coincidence number $\lambda = \wedge$. When n = 3 we use the term balanced ternary design". Thus a balanced ternary design is a collection of B blocks, each of cardinality K(K $\leq$ V), chosen from a set of size V in such a way that each of the V treatments occurs R times altogether, each of the treatments occurring once in a precisely Q_1 blocks and twice in precisely Q_2 blocks, and with incidence matrix having inner product of any two rows $\wedge_2$ is denoted by BTD (V,B,Q_1,Q_2,R,K, $\wedge_2$). It is to be noted that $Q_1 + 2Q_2 = R$. (Gupta *et al.* 1995,Sarvate and Seberry 1993).

2.2 Doubly Balanced Ternary Design

A doubly balanced *n-ary* design with parameters V,B,R,K,$\wedge$ and incidence matrix N = ((n_{ij})) is an arrangement of V treatments in B blocks,each of cardinality K(K $\leq$ V) such that (i) the i^{th} treatment appears n_{ij} times in the j^{th} block where n_{ij} can take any of the values 0,1,2,...n-1.(ii) each treatment occurs R times, and (iii) $\sum_{j=1}^{j=B} n\, ijn\, i'\, j\, nkj = \wedge 3$ for all i $\neq$ i $\neq$ k = 1,2,...V. It is to be noted that $\sum_{j=1}^{j=B} n\, ij = R$ for all i and $\sum_{j=1}^{j=V} n\, ij = K$ for all j. Thus a doubly balanced ternary design is a collection of B blocks, each of cardinality K(K $\leq$ V), chosen from a set of size V in such a way that each of the V treatments occurs R times altogether, each of the treatments occurring once in a precisely Q_1 blocks and twice in precisely Q_2 blocks, and with incidence matrix having inner product of any three rows $\wedge$ is denoted by DBTD (V,B,Q_1,Q_2,R,K,$\wedge_3$). It is to be noted that $Q_1 + 2Q_2 = R$.

2.3 Balanced Arrays (B-Arrays)

Let A be an v x b matrix, with elements 0, 1, 2, ..., s - 1. Consider the s^t ordered t-plet (x_I, x_2 , ... , x_t) that can be formed from a t-rowed sub matrix of **A** and let there be associated a positive integer ~μ(x_I, x_2, ... ,x_t) that is invariant under permutations of x_l, x_2 , ..., x_t. If

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for every t-rowed sub matrix of **A** the s^t ordered t-plets $(x_l, x_2, \ldots, x_t)$ occur $\sim\mu\ (x_l, x_2, \ldots, x_t)$ times, the matrix **A** is called a B- arrays of strength t in b assemblies with v constraints, s symbols and the specified $\mu\ (x_1, x_2, \ldots, x_t)$ parameters. The set of all permutations of $(x_1, x_2 \ldots, x_t)$ of an array of strength t in s symbols will be called the index set of the array and will be denoted by $\wedge s,t\cdot$ The array of A will be represented as the B- arrays (v, b, s, t) with index set $\wedge_{s,t}$.

3. CONSTRUCTION

Theorem 3.1 The existence of a BIB design with parameters $V=2k+1, b, r, k, \lambda$ with $b=3r-2\lambda$ implies the existence of doubly balanced ternary (DBT) design with following parameters $V=2(k+1)$, $B=b(2b-1)$, $Q_1=b^2$, $Q_2= b(b-1)/2$, $R= b(2b-1)$, $K=2(k+1)$, $\wedge_2=2r\ (b-1)+b^2$, $\wedge_3=$

$$8\binom{\lambda 3'}{2}+12\binom{\lambda 3'}{1}\binom{\lambda 2'-\lambda 3'}{1}+6\binom{\lambda 3'}{1}\binom{r'-2\lambda 2'+\lambda 3'}{1}+\binom{\lambda 3'}{1}\binom{2b-3r+3\lambda 2'-\lambda 3'}{1}+9\binom{\lambda 2'-\lambda 3'}{1}\binom{r'-2\lambda 2'+\lambda 3'}{1}$$

Proof : With the existing BIB design, a self complementary BIB design with the parameters $v' = 2(k+1)$, $b' = 2b$, $r' = b$, $k'= k+1$, $\lambda_2' = r$ can be constructed (Mitra and Mandal ,1998) in addition to $\lambda'_3 =[3r-b]/2$(pair of three treatments taken together at a time).Then BTD are constructed by taking the combination of two blocks of the self complementary design together at a time.

Hence, the number of blocks $\binom{2b}{2} = b(2b-1)$. Therefore, the total number of blocks is $B = b(2b-1)$.

The parameters *V, B,* and *K* need no explanation.

Remaining parameters are explained below :

Q_1 : Let us consider a block containing a particular treatment *x*. This will occur by taking the combination of *r* and *b-r* which is equal to

$$Q_1 = \binom{b}{1}\times\binom{b}{1} = b^2$$

Q_2 : This will occur by taking the combination of two treatments out of *r* and *0* combination out of *b-r* which is equal to

$$Q_2 = \binom{b}{2}\times\binom{b-r}{0} = b(b-1)/2$$

R : Replication number *R* for treatment *x* is $R = Q_1 + 2Q$.

Hence, $R = b(2b-1)$.

K : $2(k+1)$

$\wedge_2$: This parameter will consist of (2,2),(2,1),(1,2) and (1,1) ordered pairs of treatments.

For ordered pair (2,2), we consider 2's of the total λ's. Therefore , it is equal to $\binom{r}{2}$.

For ordered pair (2,1) and (1,2), we consider 1 λ's out of total λ's, and 1 from (r-λ) . Therefore , it is equal to $\binom{r}{1}\binom{b-r}{1}$.

For ordered pair (1,1),we consider 1 λ's and one combination of (0,0) and combination of (1,0) and (0,1) which is equal to $\binom{r}{1}\binom{r}{1}+\binom{b-r}{1}\binom{b-r}{1}$.

Thus,

$$\wedge_2 = 4r(r-1)/2 + 4r\ (b-r) + r^2 + (b-r)^2$$
$$=2r(b-1)+b^2$$

$\wedge_3$: This parameter will consist of (2,2,2),(2,2,1),(2,1,1) and (1,1,1) ordered pairs of treatments.

For ordered pair (2,2,2), we consider 2's of the total λ'_3s. Therefore , it is equal to $\binom{\lambda 3'}{2}$.

For ordered pair (2,2,1) , we consider 1 λ_3's out of total λ_3's, and 1 from $(\lambda_2-\lambda_3)$. Therefore , it is equal to 12 $\binom{\lambda 3'}{1}\binom{\lambda 2'-\lambda 3'}{1}$.

For ordered pair (2,1,1),we consider 1 λ_3's and one combination of (1,0,0) which is equal to $6\binom{\lambda 3'}{1} + \binom{r'-2\lambda 2'+\lambda 3'}{1}$

For ordered pair (1,1,1) , we consider 1 λ_3's out of total λ_3's, and (0,0,0) which is equal to $\binom{\lambda 3'}{1} + \binom{2b-3r'+3\lambda 2'+\lambda 3'}{1}$.

Another combinations of (1,1,1) in different ways provide 9 $\binom{\lambda 2'-\lambda 3'}{1}\binom{r'-2\lambda 2'+\lambda 3'}{1}$

Thus, $\wedge_3= 8\binom{\lambda 3'}{2}+12\binom{\lambda 3'}{1}\binom{\lambda 2'-\lambda 3'}{1}+6\binom{\lambda 3'}{1}\binom{r'-2\lambda 2'+\lambda 3'}{1}+\binom{\lambda 3'}{1}\binom{2b-3r'+3\lambda 2'-\lambda 3'}{1}+9\binom{\lambda 2'-\lambda 3'}{1}\binom{r'-2\lambda 2'+\lambda 3'}{1}$

Hence ,Q.E.D.

Corollary 3.1.1 The existence of a BIB design with parameters $v = b = 4t-1$, $r = k = 2t-1$, $\lambda = t-1$, implies the existence of a DBTD with parameters, $V = 4t$, $B = (4t-1)(8t-3)$, $Q_1 = (4t-1)^2$, $Q_2 = (4t-1)(2t-1)$, $R = (4t-1)(8t-3)$, $K = 4t$, $\wedge_2 = (4t-2)^2 + (4t-1)^2$, $\wedge_3 = (t-1)(23t-9)+9t^2$.

Corollary 3.1.2 The existence of a BIB design with parameters $v = 2t-1$, $b = 4t-2$, $r = 2t-2$, $k = t-1$, $\lambda = t-2$, implies the existence of a DBTD with parameters, $V = 2t$, $B = 2(2t-1)(8t-5)$, $Q_1 = (4t-2)^2$, $Q_2 = (2t-1)(4t-3)$, $R = 2(2t-1)(8t-5)$, $K = 2t$, $\wedge_2 = 4(t-1)(4t-3) + (4t-2)^2$, $\wedge_3 = (t-2)(23t-14)+9t^2$.

Example 3.1 Let us consider BIB design with parameters $v = b = 3$, $r = k = 1$, $\lambda = 0$. It implies self complementary BIB design with parameters $v' = 4$, $b' = 6$, $r' = 3$, $k' = 2$, $\lambda' = 1$. On applying Theorem 3.1,it is developed as DBTdesign (given in Table 1). The blocks given in following table represent doubly balanced ternary design.

Theorem 3.2 The existence of a BIB design with parameters $v = 2k+1$, b,r,k, λ_2 with $b = 3r-2\lambda_2$ along with self complimentary design implies the existence of DBT design with following two relationship among the parameters

(i) $B\binom{K}{2} = \Lambda_2\binom{V}{2} + Q_2 V$

(ii) $B\binom{K}{3} = \Lambda_3\binom{V}{3} + Q_2 V(V-2)$

Proof: With the existing BIB design and its complimentary, $V=2(k+1)$, $B=b(2b-1)$

$$Q_1 = \binom{b}{1}\times\binom{b}{1} = b^2,\ Q_2 = \binom{b}{2}\times\binom{b-r}{0} = b(b-1)/2$$

and the expressions of λ_2 and λ_3 are found to be

$\lambda_2=[b(V-2)]/4(V-1)$ and $\lambda_3=[b(V-4)]/8(V-1)$ and $\wedge_2=b(b+\lambda_2)-2\lambda_2$

$\wedge_3=30\lambda_2\lambda_3-12\lambda_3^2-2b\lambda_3-18\lambda_2^2+[9/2]b\lambda_2-4\lambda_3$. The above two relationship(i) and (ii) can be proved easily with $v=2k$ and $b=2r$ including the example of DBT design in Example 3.1.

Hence ,Q.E.D.

Construction of B-arrays:

Let μ_{ijk}^{fgh} denote the frequency of the t-plet in the t x b ($t \leq v$) sub-array of the b x v array in three symbols i, j, k with frequencies f, g, and h respectively such that $f+g+h=t$.

For completeness, the image method of Dey *et al*. (1972) is reproduced below: Consider a DBT design with usual parameters $(V,B,Q_1,Q_2,R,K,\wedge_2,\wedge_3)$.

Let $N(=n_{ij})$ be the incidence matrix of TGD design where

$n_{ij} = 2$ if the *jth* treatment occurs twice in the *ith* block

= 1 if the *jth* treatment occurs once in the *ith* block

= 0 , otherwise.

Evidently, N is a bxv array of symbols (0,1,2). Let any assembly of this array be denoted by a row vector $z=(z_1,z_2,\dots,z_v)$,zi= 0,1,2.

Then they defined the “images” of z as z*, given by $z^* = (z_1^*, z_2^*,\dots, z_v^*)$, $z_j + z_j^* = 3 \pmod 4$ for all i = 1,2, ..., v. Now, let M be a b x v array of “ images” of each of the assemblies of N.

Theorem 3.3

The columns of A’ when treated as assemblies give rise to a B- arrays with four symbols, 2B assemblies and strength two where A’ is given by **A’** = [N’ l M’] and A’ denotes the transpose of **A**.

Proof: The frequency of the ordered t-plet (2, 2,2) *i.e.*

μ_{012}^{003} in any t columned sub –array of N is obviously the number of blocks in which any two treatments

Table 3.1 : The number of blocks in DBT design with parameters $V = 4$, $B = 15$, $Q_1 = 9$, $Q_2 = 3$, $R = 15$, $K = 4$, $\wedge_2 = 13$, $\wedge_3 = 9$

B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}	B_{11}	B_{12}	B_{13}	B_{14}	B_{15}
1	1	1	1	1	2	2	1	1	2	1	1	1	1	1
2	3	2	1	1	3	2	2	2	3	3	2	2	2	1
4	4	3	3	2	4	3	3	2	3	3	3	3	2	2
4	4	4	4	4	4	4	4	4	4	4	4	3	3	3

occur together and is therefore equal to $\binom{\lambda 3'}{2}$. The frequency of the other t-plet (2,2,1) *i.e.*

$$\mu_{012}^{021} = \binom{\lambda 3'}{1}\binom{\lambda 2' - \lambda 3'}{1}.$$

Proceedings like this,

$$\mu_{012}^{021} = \binom{\lambda 3'}{1}\binom{r - 2\lambda 2' + \lambda 3'}{1} + \binom{\lambda 2' - \lambda 3'}{1}\binom{\lambda 2' - \lambda 3'}{1}$$

$$\mu_{012}^{030} = b$$

$$\mu_{012}^{102} = \binom{\lambda 2' - \lambda 3'}{1}\binom{\lambda 2' - \lambda 3'}{1}$$

$$\mu_{012}^{111} = \binom{\lambda 2' - \lambda 3'}{1}\binom{r - 2\lambda 2' + \lambda 3'}{1}$$

$$\mu_{012}^{120} = \binom{r' - 2\lambda 2' + \lambda 3'}{1}\binom{2b - 3r' + 3\lambda 2' - \lambda 3'}{1}$$

$$+ \binom{r' - 2\lambda 2' + \lambda 3'}{1}\binom{r' + 2\lambda 2' - \lambda 3'}{1}$$

$$\mu_{012}^{210} = \binom{r' - 2\lambda 2 + \lambda 3'}{1}\binom{2b - 3r' + 3\lambda 2' - \lambda 3'}{1}$$

Since the assemblies of M are “images” of those of N, it follows that in any t-columned sub-array of M, the frequency of the ordered t-plets will be corresponding to N *i.e.*

The frequency of the ordered t-plet (1,1,1) *i.e.*

μ_{123}^{300} in any t columned sub –array of N is obviously the number of blocks in which any three treatments occur together and is therefore equal to $\binom{\lambda 3'}{2}$. The frequency of the other t-plet (1,1,2) i.e

$$\mu_{123}^{021} = \binom{\lambda 3'}{1}\binom{\lambda 2' - \lambda 3'}{1}.$$

Proceedings like this,

$$\mu_{123}^{012} = \binom{\lambda 3'}{1}\binom{r - 2\lambda 2' + \lambda 3'}{1} + \binom{\lambda 2' - \lambda 3'}{1}\binom{\lambda 2' - \lambda 3'}{1}$$

$$\mu_{123}^{003} = b$$

$$\mu_{123}^{021} = \binom{\lambda 2' - \lambda 3'}{2}$$

$$\mu_{123}^{111} = \binom{r' - 2\lambda 2' + \lambda 3'}{1}\binom{2b - 3r' + 3\lambda 2' - \lambda 3'}{1}$$

$$\mu_{123}^{021} = \binom{r' - 2\lambda 2' + \lambda 3'}{1}\binom{2b - 3r' + 3\lambda 2' - \lambda 3'}{1} +$$

$$\binom{r' - 2\lambda 2' + \lambda 3'}{1}\binom{r' + 2\lambda 2' - \lambda 3'}{1}$$

$$\mu_{123}^{012} = \binom{r' - 2\lambda 2' + \lambda 3'}{1}\binom{2b - 3r' + 3\lambda 2' - \lambda 3'}{1}$$

Therefore, in the whole array **A** the frequency of all ordered t-plets of the treatments are given by μ_{0123}^{0300} and μ_{0123}^{0030} in any t columned sub –array of N is obviously the number of blocks in which any two treatments occur together and is therefore equal to b The frequency of the other t-plet will be same as described above .

$$\mu_{0123}^{0120} = \binom{\lambda 3'}{1}\binom{\lambda 2' - \lambda 3'}{1}$$

Proceedings like this,

$$\mu_{0123}^{0210} = \binom{\lambda 3'}{1}\binom{r - 2\lambda 2' + \lambda 3'}{1} + \binom{\lambda 2' - \lambda 3'}{1}\binom{\lambda 2' - \lambda 3'}{1}$$

$$\mu_{0123}^{1020} = \binom{\lambda 2' - \lambda 3'}{1}\binom{\lambda 2' - \lambda 3'}{1}$$

$$\mu_{0123}^{1110} = \binom{\lambda 2' - \lambda 3'}{1}\binom{r - 2\lambda 2' + \lambda 3'}{1}$$

$$\mu_{0123}^{1200} = \binom{r' - 2\lambda 2' - \lambda 3'}{1}\binom{2b - 3r' + 3\lambda 2' - \lambda 3'}{1} +$$

$$\binom{r' - 2\lambda 2' + \lambda 3'}{1}\binom{r' - 2\lambda 2' + \lambda 3'}{1}$$

$$\mu_{0123}^{2100} = \binom{r' - 2\lambda 2' - \lambda 3'}{1}\binom{2b - 3r' + 3\lambda 2' - \lambda 3'}{1}$$

Since the assemblies of M are “images” of those of N, it follows that in any t-columned sub-array of M, the frequency of the ordered t-plets will be corresponding to N .

The frequency of the other t-plet (1,1,2) *i.e*

$$\mu_{0123}^{0210} = \binom{\lambda 3'}{1}\binom{\lambda 2' - \lambda 3'}{1}$$

Proceedings like this,

$$\mu_{0123}^{0012} = \binom{\lambda 3'}{1}\binom{r-2\lambda 2'+\lambda 3'}{1} + \binom{\lambda 2'-\lambda 3'}{1}\binom{\lambda 2'-\lambda 3'}{1}$$

$$\mu_{0123}^{0021} = \binom{\lambda 2'-\lambda 3'}{2}$$

$$\mu_{0123}^{0111} = \binom{r'-2\lambda 2'+\lambda 3'}{1}\binom{2b-3r'+3\lambda 2'-\lambda 3'}{1}$$

$$\mu_{0123}^{0021} = \binom{r'-2\lambda 2'+\lambda 3'}{1}\binom{2b-3r'+3\lambda 2'-\lambda 3'}{1} +$$

$$\binom{r'-2\lambda 2'+\lambda 3'}{1}\binom{r'-2\lambda 2'+\lambda 3'}{1}$$

$$\mu_{0123}^{0012} = \binom{r'-2\lambda 2'+\lambda 3'}{1}\binom{2b-3r'+3\lambda 2'-\lambda 3'}{1}$$

Thus, A is a four symbol B- arrays of strength three with index set $\wedge_{4,3}$. The frequency of all other t-plets combinations are zero.

Hence, Q.E.D.

4. Illustrative Example

Example 4.1

Consider the incidence matrix of the DBT design with given blocks in Example3.1 and applying image method of Dey *et al*.(1972) ,we get X is a

B- arrays with parameters (v= 4, b = 30, s =4, t=3) with index set $\wedge_{4,3}$.

$$\begin{array}{cc} & \begin{array}{cc} N' & M' \end{array} \\ X = & \left(\begin{array}{c|c} 1102 & 2231 \\ 1012 & 2321 \\ 1111 & 2222 \\ 2011 & 1322 \\ 2101 & 1232 \\ 0112 & 3221 \\ 0211 & 3122 \\ 1111 & 2222 \\ 1201 & 2132 \\ 0121 & 3212 \\ 1021 & 2312 \\ 1111 & 2222 \\ 1120 & 2213 \\ 1210 & 2123 \\ 2110 & 1223 \end{array}\right) \end{array}$$

The frequency of other treatment combinations of strength 3 is zero .

5. Applications

Deleting three blocks given in Example 3.1, it can be used for conducting intercropping experiments when the intercrops are subdivided into various groups based on agronomic practices. We construct designs for experiments where each plot consists of two main crops and eight intercrops in such a way that each of these intercrops is selected from a group of intercrops following Rao and Rao (2001).

Now, let us consider an intercropping experiments using two main crops and eight intercrops where the intercrops are divided into four groups S_1,S_2,S_3,S_4 with two in each group *viz*., S_1 = [1,2], S_2 =[3,4], S_3 = [5,6], S_4 = [7,8] .Let us designate the symbols 0, 2 of first row of BTD design with intercrops 1, 2 of S_1 , second row with intercrops 3, 4 of S_2, third row with intercrops 5,6 of S_3 and fourth row of intercrops 7,8 of S_4 .Taking into the consideration the column of the array as the plots of the intercropping experiments in addition to two main crops in each plot, the resulting intercropping experiments will consist of the following twelve plots on the basis of the blocks given in the Example 3.1.

(m_1,m_2,5,8) ; (m_1,m_2,3,8) ; (m_1,m_2,2,3) ; (m_1,m_2,2,5); (m_1,m_2,1,8); (m_1,m_2,1,4)

(m_1,m_2,4,5) ; (m_1,m_2,1,6) ; (m_1,m_2,3,6) ; (m_1,m_2,6,7); (m_1,m_2,4,7); (m_1,m_2,2,7)

This layout of the intercropping experiment is found to be superior having one main crop and six intercrops in a six plots rather than that of Sharma *et al.* (2013).

In the context of an actual example of intercropping experiment, Takim (2012) have used the different mix-proportions and planting patterns of maize (*Zea mays* L.) and cowpea (*Vigna unguiculata* L.) for the comparisons of sole cropping of each crop during 2010 and 2011 growing seasons under the southern Guinea savanna conditions in Nigeria. The experiment comprised of 6 treatments: sole maize (51,282 plants ha^{-1}), sole cowpea (61,538 plants ha^{-1}) and 4 maize – cowpea intercropping mix-proportion: 100 maize: 100 cowpea, 50 maize: 50 cowpea ,60 maize:40 cowpea and 40 maize:60 cowpea using randomized complete block design with three replications. Evaluation of the intercropping patterns was performed on basis of several intercropping indices.The study revealed that the mix-proportion of 50 maize:50 cowpea gave a similar grain yield compared to other intercropped plots.The study also revealed that intercropping systems could be an eco-friendly approach for reducing weed problems through non-chemical methods, mix-proportion of 50 maize:50 cowpea planted on alternate rows could be a better intercropping pattern.

In another example of intercropping experiment, Pandey *et al.* (2003) have studied the effect of maize (*Zea mays* L.) based intercropping system on maize yield as main crop and six intercrops *viz.*, pigeonpea, sesamum, groundnut, blackgram,turmeric and forage *meth* by conducting an experiment during the rainy seasons of 1998 and 1999 at the research farm of Rajendra Agricultural University, Pusa, Samastipur (Bihar). The experiment consisted of six intercrops with one main crops was conducted in randomized complete block design with four replications. Maize was grown at a spacing of 75 cm. Showing dates in sole as well as in intercropping was on 26 and 22 June respectively in the first and second year of experimentation. One row of pigeon pea at a distance of 75 cm and 2 rows of other intercrops at 30 cm distance were accommodated between two rows of maize. The intra row spacing of 30, 30, 10, 15, 10, and 15 cm were maintained by thinning for six intercrops.

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