# Evolution of initial mass function in Young Massive Star Clusters (YMCs)

# Abisa Sinha, Tanuka Chattopadhyay and Asis Kumar Chattopadhyay

Department of Statistics, Ballygunge Science College Campus Calcutta University, 35, Ballygunge Circular Road, Kolkata-700 019

#### ABSTRACT

The stellar initial mass function (IMF) is critical to models of star formation on all scales ranging from individual stars up to entire galaxies. The present study deals with the evolution of stellar initial mass function in young massive clusters (YMCs) including collision and evaporation of the fragments under various physical considerations together with the accretion of residual gas on them. Investigations have been carried out to identify the initial conditions under which it converges to stellar mass function in globular clusters and the corresponding time to attend this. It is found that if YMCs are really precursors of globular clusters then evaporation due to collisions has to be weakened after 2-4 Myr and after that purely stellar dynamical models should dominate the cluster evolution.

Keywords : Initial mass function; dynamical evolution; star clusters

### 1. Introduction

Young Massive Clusters (YMCs) are big compact ensembles of stars (  $10^4$ - $10^6 M_{\odot}$ ) having ages of few mega-years. They have been observed in our Galaxy as well as in external galaxies of the Local Group (Zwart et.al., 2010). The evolution of star clusters generally consists of three phases. In the first phase, star formation takes place and the cluster contains significant amount of gas. In the second phase, the cluster is more-or-less gas free but mass loss plays a dominant role in the dynamical process. In the final phase, purely stellar dynamical model dominates the long term evolution of the cluster. The transition from phase 1 to 2 is assumed to be of the order of 3Myr, which is the time of formation of the first supernova (Eggleton 2006) and the time from phase 2 to 3 is in between 100 M yr and 1 Gyr depending on various initial conditions like cluster initial mass, radius, density profile and stellar mass function (Zwart et.al., 2010).

YMCs appear to form with a cluster mass function, which is a truncated power law with index - 2 (Zhang and Fall 1999; Bik *et al.*, 2003; McCrady and Graham 2007) having an indication of truncation at around  $10^5$  $M_{\odot}$  in normal galaxies whereas it is greater than  $10^6$  $M_{\odot}$  in star burst galaxies. Their dynamical ages are much smaller (Zwart *et al.*, 2010) than the present ages which indicate that these systems are bound systems.

The evolution of these clusters in the first phase is not clearly understood (Elmegreen 2007; Price and Bate 2009). This incorporates many uncertainties in the cluster properties. It is speculated that Antennae galaxies will finally evolve into an elliptical galaxy (Zhang and Fall 1999; Zwart et al., 2010) and its young massive stellar populations will evolve into old globular clusters seen in other elliptical galaxies. So it raises the inevitable question, "whether the YMCs are young analogue of old globular clusters"? Now by definition, globular clusters, as we know, are roughly spherical systems with large numbers of old metal poor stars and are observed in galactic halos. At present, the number of YMCs observed is very small. The spatial distribution of YMCs in our Galaxy lies along the disc like open clusters but in massradius diagram they are closely related to globular clusters (GCs) (Figer et al., 1999; Pfalzner 2009; Harayama et al., 2008; Figer et al. 2006; Davies et al. 2007; Mengel and Tacconi 2007). Also they are metal rich than GCs of our own Galaxy (Zwart et al., 2010). So it is quite unreasonable to reject YMCs as precursors of globular clusters only on the basis of such dissimilarity and it is possible that any massive cluster more than a few tens of dynamical times old can have a smooth, roughly spherical appearance, irrespective of metallicity or location. To answer such speculation, one requires a detailed knowledge of various cluster properties to predict how a cluster will evolve. One such criterion is the form of initial stellar mass function. Others involve uncertainties about the cluster's orbit in the host galaxy and its ability to survive destructive dynamical effects.

In the present work, an evolutionary model has been developed for YMCs in order to determine the initial conditions under which YMC can evolve into a system like globular cluster and the corresponding time involved in such process. In Section 2, the mathematical model has been developed involving coalescence and

E-mail : akcstat@caluniv.ac.in

evaporation of the fragments as well as accretion of the residual gas on the fragments during the period after which gas dispersal occurs. In section 3, numerical values of the parameters have been discussed while Section 4 includes results and interpretation.

### **2 THE MATHEMATICAL MODEL**

### 2.1 Fragmentation and Mass distribution

While considering the IMF of stars, several authors (Elmegreen and Mathieu 1983; Wolf and Vadimir 1986; Larson 1973) have considered the application of the random fragmentation theory. Random fragmentation of a line into n parts and its application in assessing the randomness of radioactive disintegration and cosmicray events were considered by Feller (1980). Elmegreen and Mathieu (1983) studied the form of the mass spectrum by considering random fragmentation using a Monte Carlo simulation for a time-independent model suggested by Larson (1973). They assumed an initial Gaussian distribution of the number of fragments as well as their masses, but in their work, they did not introduce the effect of the time interval between two successive fragmentations, which is also a random variable. Chattopadhyay et al. (2003) considered this effect in the following manner. If a line of length 1 is divided at random into N\_F parts, the average number of fragments will exceed the value x, given by Feller (1980)

$$N_F \left( 1 - \frac{\chi}{1} \right)^{N_{F-1}} \tag{1}$$

Following the above argument, if  $N_F$  is the total number of fragments formed within a given time interval  $t_1$  (say) after a fragmentation step in the hierarchy, then the probability that the time elapsed between successive fragmentations will not exceed *t* is given by

$$P(t, N_F, t_1) = 1 - (1 - t/t_1)^N F$$
(2)

Previously, Auluck and Kothari (1965) considered only  $N_F$  to be random in the above expression, but Chattopadhyay *et al.* (2003) have also considered time t a random variable because the time interval between two successive fragmentations also has to be random. According to Feller (1980), the probability of the occurrence in a run of duration t of exactly n events is given by the Poisson formula

$$W_n(t) = e^{-\lambda t} \frac{(\lambda t)^n}{n!}$$
(3)

Here, the distribution of inter-occurrence time is given by

$$P(t) = 1 - e^{-\lambda t} \tag{4}$$

In the above expression, the parameter  $\lambda$  can be estimated by the reciprocal of the average time  $(\bar{t})$ 

between successive fragmentations, *i.e.*,  $\hat{\lambda} = 1/\bar{t}$ 

The average time of fragmentation (Chattopadhyay *et al.* 2003) is  $\overline{t} \le y / n$ , where *y* is the maximum time of successive fragmentation steps and n is the number of fragmentation steps. Further,

$$n \le (\log m_f + \log m_{min}) / (\log N_F) \tag{5}$$

where  $m_f$  is the mass of the parent cloud and  $m_{min}$  is the minimum mass of a fragment. Here, we consider the function of m masses along with t *i.e.* N(m; t), number of fragments formed after collision of particles as of our point of interest and what it becomes after coalescence and evaporation.

### 2.2 Modelling of the fragmentation procedure

Previously, we pointed out that star clusters consist of initial, intermediate and final stages of stellar evolution, where the initial phase consists of fragmentation and intermediate one involves dynamical interaction and accretion. In the present work, we have considered massive spherically symmetric gas cloud which has undergone hierarchical fragmentation and is in the phase of evolution through dynamical interaction and accretion of residual gas accompanied by evaporation.

Evaporation of small stars from a star cluster is a very significant phenomenon as it changes the shape of the stellar mass function (He'non 1969; Chernoff and Weinberg 1990; Baumgardt and Makino 2003). The stellar mass function has been observed for globular clusters as well as for many ensembles e.g. stellar associations, dense clusters, open clusters (De Marchi et.al., 2007; Richer et.al., 1991; Marchi et.al.) which have a flatter form in the low mass range than a typical Salpeter(1955) type. The flatter form results in a lowmass-to-light ratio which can be explained by the dynamical evolution involving evaporation (Kruijssen 2008; Kruijssen and Mieske 2009; Andersen et.al., 2009). So, in the present work, dynamical evaporation has been included for the disintegration of the cloud fragments undergoing evolutionary process.

It has been discussed that during fragmentation of a massive cloud sufficient gas is left in the cloud which will subsequently accrete on the fragments moving randomly within the parent cloud, depending upon their masses. The gas depletion starts when first supernova explodes which is of the order of few M yrs. So, the present model includes both accretion and evaporation during its dynamical evolution together with collisions among themselves. Murray and Lin (1989 a,b) have shown that thermal instability in a proto globular cluster cloud (PGCC) is comparable with the cooling time scale

 $\tau_c$  where,

$$\tau_c = \frac{3}{2} \frac{kT}{n\Lambda(x,t)} \tag{6}$$

For a cloud of mass  $1.6 \times 10^6 M_{\odot}$ , density  $n = 270 \ cm^{-3}$ ,  $\tau_c = 0.9 \ \tau_d$  *i.e.*  $\tau_c < \tau_d$ . They found for such a cloud after  $1.58 \times 10^{13} sec$  (~  $\tau_c$ ), density fluctuation  $\delta \sigma$ 

 $\frac{\sigma \rho}{\rho}$  has increased by less than a factor of 2 and this leads to the formation of a cold dense shell. The time scale of gravitational instability  $\tau_g$  of this cold dense shell is less than  $\tau_c$  if  $\left(\frac{\rho}{m_H}\right)$  of the shell  $\geq 1321 \text{ cm}^{-3}$ ,

so that the entire process of fragmentation occurs in a time scale which is shorter than the dynamical time scale *i.e.*  $\tau_c + \tau_g \leq \tau_d$ . Also, Nakano (1966) has shown that the ratio of collision time  $t_{coll}$  and dynamical time  $t_f$  (*i.e.* the free fall time) is,

$$\frac{t_{coll}}{t_f} \simeq (\pi f)^{-1} (\frac{m}{M})^{1/3} (\frac{\rho}{\rho_b})^{7/6}$$
(7)

For  $M = 10^5 - 10^6 M_{\odot}$ ,  $m = M_{\odot}$ ,  $\rho = \rho b = 25$ , f=0.1,  $t_{coll}=t_f \approx 2.9$  to 1.4. In the present problem, we have considered YMCs whose number density is similar to their parent cloud *viz*.  $10^4 cm^{-3}$ . So, the dynamical time is of the order of  $10^5$  years, such that collisions do not occur during fragmentation phase (~  $10^5$  year), on the basis of the above discussion. We have considered the evolution of YMCs in three phases : (i) The fragmentation phase (~  $10^5$  year), (ii) the collision, evaporation, accretion phase (~ M yr) (iii) final stage of evolution. In the previous work (Chattopadhyay *et.al.* (2011), hereafter *C*11), we have considered the fragmentation of molecular clouds whose mass is comparable to the mass of YMCs and computed the resulting mass spectrum along the line of sight in the core as well as in the envelope to search for an existence of mass segregation, if any, having primordial origin. In the present scenario, we have used the mass spectrum of those clouds as initial inputs and constructed a model involving dynamical interaction, evaporation and accretion of the residual gas.

Let N(m, t) be the number of fragments in the mass range (m, m + dm), which is increased by coalescence of fragments of masses m' and m - 'm and is decreased when it escapes from the star cluster as a result of collision with the remaining fragments. The escape rate is given by

$$(\frac{dM}{dt})_{dis} \quad \chi(m),$$
 (8)

(Kruijssen 2009), where

$$\left(\frac{dM}{dt}\right)_{dis} = -\frac{M}{t_{dis}} = -\frac{M^{1-\gamma}}{t_0} \tag{9}$$

M is the mass of parent cloud,  $\gamma$  is a parameter whose value is roughly around 0:6 and  $t_0$  is the dissolution time. For computing the function  $\chi$  (*m*), the values of the parameter  $c_1$  and < m > are required, where  $c_1$  denotes the ratio of mean speed squared to the central escape velocity squared and < m > is the average mass of the fragments. The total mass of an YMC is of the order of  $10^6 M_{\odot}$ , then for such a cloud dividing the total mass by total number of fragments, the value of < m > is roughly around 0.42  $M_{\odot}$ . Since, globular cluster's age is of the order of Gyr and we are studying whether YMCs can give rise to old globular clusters in course of their evolution, we consider the dissolution time t0 to be of the order of Gyr, comparable with the age of the older globular clusters. So, the resulting model now takes the form.

$$\frac{\partial N(m,t)}{\partial t} + \frac{\partial}{\partial m} [A_c(m)N(m,t)] \\ = \frac{1}{2} \int_{m_{min}}^{m-m_{min}} \alpha(m',m-m')N(m',t)N(m-m',t)dm' + (\frac{dM}{dt})_{dis} \chi(m)$$
(10)

where  $\alpha$  (*m*,*m*') is the collision or impact parameter between masses *m* and *m*', with *m*<sub>min</sub> and *m*<sub>max</sub> being the minimum and maximum masses of a fragment, while  $A_c(m)$  is the accretion rate on a fragment of mass *m*.

#### 3. Numerical values of the Parameters

#### 3.1 The Form of Impact Parameter

Field and Saslaw (1965) have used the form of the cloud collision as,

$$\alpha(m, m') = 2\sqrt{2}\pi r^2 v \frac{m + m'}{(mm')^{1/2}}$$
(11)

if the clouds have the same density and are in a condition of equipartition of kinetic energy. We use this expression in our calculation where v and r are the mean speed and radius of the fragments. Here,

$$v = \sqrt{\frac{GM}{R}}$$
(12)

where *M* and *R* are the mass and size of the parent cloud. In the previous work (*C*11), we have considered the fragments of massive clouds of size 15pc and masses varying from  $10^5 M_{\odot} - 10^6 M_{\odot}$ . Here also, we have used those values for M and R respectively. For the mean size of a fragment, *r*, we have assumed that the very onset of collapse of an average solar type fragment starts its life with a size of the order of  $10^5 R_{\odot}$ .

#### **3.2 Accretion Rate**

Spitzer (1987) considered the accretion rate by an object of mass m, surrounded by extended gas as,

$$\frac{dm}{dt} = \frac{4\pi \left(Gm\right)^2 \rho_1}{v^3} \tag{13}$$

where,  $\rho_1$  and v are the density and velocity of the accreting material at infinity. In an YMC, the number density varies from  $10^4 - 10^5 cm^{-3}$  from outer surface to the core (Bally *et.al.*(1987; 88)). So, for a number density from  $5 \times 10^4 cm^{-3}$  to  $10^5 cm^{-3}$  and v, found from equation (7) for M ranging from  $10^5 M_{\odot} - 10^6 M_{\odot}$ , the values for  $m = 1M_{\odot}$  are  $3.4 \times 10^{-9}$  to  $9 \times 10^{-9} M_{\odot} yr^{-1}$ .

In the above calculation, he assumed the accreting material has infinite mean free path whereas in practice the accreting material will undergo frictional resistance which slows down the rate of accretion and instead of the above expression, we considered the rate as:

$$\frac{dm}{dt} = Bm^{(1-p)/3},$$
(14)

(Basu and Bhattacharya 1984), where *B* and *p* are constants. For inelastic collision,  $\frac{d^2M}{dt^2} > 0$  which gives

p < 1 and since p > 0 ,  $0 If <math display="inline">\gamma' = \frac{1 - p}{3}$  then

 $0 < \gamma' < 0.33$ . So, the rate of accretion is,

$$\frac{dm}{dt} = Bm^{\gamma'}, \quad 0 < \gamma' < 0.33 \tag{15}$$

For the present model the rate is normalized for  $m = 1M_{\odot}$  with the value in equation (8) giving,  $3.4 \times 10^{-9} \le B \le 9 \times 10^{-9}$ .

#### 3.3 Minimum and Maximum masses of a fragment

The minimum mass, as a result of fragmentation of a big molecular cloud, is close to  $10^{-2}M_{\odot}$  or less (Silk 1977; Kanjilal and Basu 1992). The maximum mass of a star in a cluster is found to be of the order  $10^{2}M_{\odot}$ 

(Faustini *et.al.*, 2009; Martins *et.al.*, 2005; Maschberger and Clarke 2008; Parkar and Goodwin 2007). So, for  $m_{min}$  and  $m_{max}$  the values considered are around  $0.05M_{\odot}$  and  $100M_{\odot}$ , respectively. The initial values of the parameters are listed in table 1.

Table 1: Initial values of the parameters

|                            | -                   |
|----------------------------|---------------------|
| Parameter                  | Value               |
| $c_1$                      | 0.02                |
| $< m > (M_{\odot})$        | 0.42                |
| r(pc)                      | 0:231               |
| R(pc)                      | 15                  |
| $t_0$ (year)               | $10^{9}$            |
| γ                          | 0.62                |
| γ                          | 0.2                 |
| В                          | $3.7 	imes 10^{-9}$ |
| $m_{_{min}}(M_{\bigcirc})$ | 0.05                |
| $m_{max}(M_{\odot})$       | 100                 |
| $\alpha_1$                 | from <i>C</i> 11    |
| $\alpha_2$                 | from <i>C</i> 11    |
| M                          | from <i>C</i> 11    |
| m <sub>c</sub>             | from C11            |

<sup>1</sup>Mackey and Gilmore(2003)

<sup>2</sup>McLauglin and Van Der Marel(2005)

<sup>3</sup>Perina et al. (2009)

<sup>4</sup>Barmby *et al.* (2009)

<sup>5</sup>Mengel and Tacconi(2007)

### **4 RESULTS AND DISCUSSION**

It has been discussed in the previous section that the evolution of YMCs are divided into three phases. In the initial phase, there is only fragmentation of the parent cloud until the opacity of a fragment is sufficient enough to halt further fragmentation and a phase change from isothermal to adiabatic condition takes place in the system. In the previous work (C11), we have described fragmentation of clouds whose masses are of the order of YMCs modelling a random fragmentation scenario under various initial conditions both in the core as well as in envelopes of the clouds. We found the form of the mass spectrum, maximum mass of the fragments  $(m_{max})$ , critical mass ( $m_c$  where a turnover of the mass spectrum occurs) as a function of cloud masse, size, efficiency of star formation etc. The mass spectrum are well fitted by segmented power laws of the forms:

Evolution of Initial Mass Function in Young Massive Star Clusters

$$\xi_{IMF}(m) = \frac{dN}{dm} = \begin{cases} A_1 m^{-\alpha_1}; & m_{min} < m \le m_c \\ A_2 m^{-\alpha_2}; & m_c < m \le m_{max} \end{cases}$$
(16)

when  $A_1$  and  $A_2$  are solved to make:

$$\hat{A}_1 = \hat{A}_2 m_c^{\alpha_1 - \alpha_2} \\ \hat{A}_2 = \left[\frac{m_c^{\alpha_1 - \alpha_2}}{1 - \alpha_1} (m_c^{1 - \alpha_1} - m_{min}^{1 - \alpha_1}) + \frac{1}{1 - \alpha_2} (m_{max}^{1 - \alpha_2} - m_c^{1 - \alpha_2})\right]^{-1}$$
(17)

The values of  $m_{min}$ ,  $m_{max}$ ,  $m_c$ ,  $\alpha_1$  and  $\alpha_2$  are taken from C11. In the present problem, we have considered those outputs as our initial inputs since we are studying the evolution of those clouds starting at the end of phase 1 and evolving through the phase 2, *i.e.* the phase of collisions, evaporation and accretion, unless the mass spectrum converges to that of an old globular star cluster, *i.e.*,  $\alpha_2 \sim 2.0$  (Bastian *et. al.*, 2010). At that point,  $\alpha_1$ and  $\boldsymbol{\alpha}_2$  are compared together with total time elapsed (viz.  $\alpha'_1$ ,  $\alpha'_2$ , T). These are listed in table 2 under various physical conditions. Change of mass spectrum in phase 2 for a typical YMC is shown in figure 2. It is clear from table 2 that if YMCs are really precursor of globular clusters, then evaporation due to collision has to be weakened after 2 - 4Myr and after which purely stellar dynamical models should dominate the cluster evolution.

This is evident from column 9 of table 2, where the time elapsed for convergence of the initial mass function to the observed globular cluster initial mass function (Bastian et.al., 2010) is of the order of few Myr both in the core as well as in the envelope of a particular cluster. On the other hand we can conclude that while fragmentation procedure plays a dominant role in the phase 1 (C11), evaporation is the primary mechanism in evolution of the stellar mass function in massive star clusters when they are in phase 2. Table 2 shows that primordial mass segregation is present in the globular clusters in their second phase of evolution but the variation is much weaker than those in phase 1 if we compare the 2 indices at the end of phase 2 (column 12) with those at the end of phase 1 (column 4)(listed from our previous work C11).

Table 2: Segmented power law fits to YMCs at the end of phase 2 for a minimum mass at  $0.05 M_{\odot}$ 

|  |                   |            |      |                |                  |                        |      |              |                 | $\cup$ |      |
|--|-------------------|------------|------|----------------|------------------|------------------------|------|--------------|-----------------|--------|------|
| Name                                   | Mass              | $\alpha_1$ | α2   | m <sub>c</sub> | m <sub>max</sub> | v                      | b    | Time elapsed | m' <sub>c</sub> | α'1    | α'2  |
|  | $(M_{\odot})$     |            |      | $(M_{\odot})$  | $(M_{\odot})$    | ( <i>pc/yr</i> )       | (pc) | (Myr)        | $(M_{\odot})$   |        |      |
| NGC 330 <sup>1, 2</sup>                | 10 <sup>5.8</sup> | -0.12      | 2.49 | 0.25           | 132.41           | 13.68×10 <sup>-6</sup> | 2    | 3            | 0.55            | 0.817  | 1.82 |
|  |                   | -0.32      | 2.73 | 0.22           |                  |                        | 12   | 4            |                 | -0.887 | 1:80 |
| M31 V db0 <sup>3</sup>                 | 10 <sup>5</sup>   | -0.12      | 2.45 | 0.25           | 139.85           | 5.45×10 <sup>-6</sup>  | 2    | 3            | 0.55            | -0.432 | 1.88 |
|  |                   | -0.33      | 2.43 | 0.26           |                  |                        | 12   | 3            |                 | -0.582 | 1.97 |
| <i>M</i> 31 <i>B</i> 2570 <sup>4</sup> | 10 <sup>5</sup>   | -0.02      | 2.53 | 0.27           | 138.28           | 5.45 ×10 <sup>-6</sup> | 2    | 3            | 0.55            | -0.530 | 2.06 |
|  |                   | -0.28      | 2.77 | 0.26           |                  |                        | 12   | 4            |                 | -0.617 | 2.05 |
| <i>LMC NGC</i> 2164 <sup>1,2</sup>     | 10 <sup>5.2</sup> | -0.26      | 2.25 | 0.26           | 141.05           | $6.85 	imes 10^{-6}$   | 2    | 2            | 0.55            | -0.712 | 2.00 |
|  |                   | -0.05      | 2.27 | 0.25           |                  |                        | 12   | 2            |                 | -0.687 | 2:02 |
| <i>LMC NGC</i> 2214 <sup>1,2</sup>     | 10 <sup>5.4</sup> | -0.01      | 2.28 | 0.26           | 140.33           | $8.63 	imes 10^{-6}$   | 2    | 2            | 0.55            | -0.502 | 1.94 |
|  |                   | -0.18      | 2.55 | 0.23           |                  |                        | 12   | 3            |                 | -0.761 | 1.89 |
| NGC 4038 S23 <sup>5</sup>              | 10 <sup>5.4</sup> | -0.25      | 2.43 | 0.23           | 123.01           | $8.63 	imes 10^{-6}$   | 2    | 3            | 0.55            | -0.798 | 1.76 |
|  |                   | -0.06      | 2.62 | 0.27           |                  |                        | 12   | 3            |                 | -0.789 | 1.98 |
| NGC 4038 S1 <sup>5</sup> <sub>5</sub>  | 10 <sup>5.6</sup> | -0.15      | 2.57 | 0.26           | 113.88           | $10.87\times10^{-6}$   | 2    | 3            | 0.55            | -0.588 | 1.89 |
|  |                   | -0.06      | 2.62 | 0.27           |                  |                        | 12   | 3            |                 | -0.610 | 2.04 |
| NGC4038 S2 <sup>5</sup> 1              | 106               | -0.24      | 2.49 | 0.24           | 133.37           | $10.87\times10^{-6}$   | 2    | 3            | 0.55            | -1.43  | 1.74 |
|  |                   | -0.27      | 2.49 | 0.25           |                  |                        | 12   | 3            |                 | -1.47  | 1.74 |

Sinha et al.



Fig. 1: The form of initial mass function for various YMCs at the end of phase 2



Fig. 2: Evolution of spectral index in phase 2 for a typical YMC NGC 330

## REFERENCES

- Andersen, M., Zinnecker, H., Moneti, A., McCaughrean, M.J., Brandl. 2009. "The Low-mass initial mass function in the 30 Doradus Starburst Cluster", 2009. *Astrophysical Journal*, 707, 1347.
- Auluck, F. C. and Kothari, D. S. 1965. "Random Fragmentation", 1965, Zeitschrift fur Astrophysik, 63, 9.
- Bally, J., Stark, A.A., Wilson, W.and Henkel, C.1988. "Galactic Center Molecular Clouds. I. Spatial and Spatial-Velocity Maps", 1988, Astrophysical Journal, 324, 223.
- Bally, J., Stark, A. A., Wilson, W. and Henkel, C.1987. "Galactic Center Molecular Clouds. II. Spatial and Spatial-Velocity Maps" 1987, Astrophysical Journal, 65, 13.
- Barmby, P., Perina, S., Bellazzini, M., Cohen, J.G. and Hodge, P.W. 2009. "A Hubble Space Telescope/ WFPC2 Survey of Bright Young Clusetrs in M31. III. Structural Parameters", 2009. Astronomical Journal, 138,1667.
- Bastian, N., Covey, K.R. and Meyer, M.R. 2010. "A Universal Stellar Initial Mass Function? A Critical Look at Variations", *Annual Review of Astronomy and Astrophysics*, 2010, 48, 339.
- Basu, B. and Bhattacharya, T. 1984. "Gravitational collapse of cosmic gas clouds and formation of star clusters", *Astrophysics Space Sci.*, 105,85.
- Baumgardt, H. and Makino, J. 2003. "Dynamical evolution of star clusters in tidal fields", 2003, Monthly Notices of the Royal Astronomical Society, 340,227.
- Bik, A., Lamers, H.J.G.L.M., Bastian, N., Panagiam N. and Romaniello, M. 2003. 'Clusters in the inner spiral arms of M51: the cluster IMF and the formation history.", 2003, Astronomy and Astrophysics, 397,473.
- Chattopadhyay, T., Chattopadhyay, A.K. and Sinha, A. 2011, "Modeling of the Initial Mass Function Using The Metropolis-Hastings Algorithm", *Astrophysical Journal*, 736,152.
- Chernoff, D.F. and Weinberg, M.D.1990. "The Origin of IRS 16: Dynamically Driven In-Spiral of a Dense Star Cluster to the Galactic Center?" *Astrophysical Journal*, 351,121.
- Davies, B., Figer, D.F., Kudritzki, R.P, MacKenty, J., Najarro, F. and Herrero, A. 2007. "A Massive Cluster of Red Supergiants at the base of the Scutum-Crux Arm", 2007, *Astrophysical Journal*, 671, 781.
- De Marchi G., Paresce F., and Pulone L. 2007."Why Haven't Loose Globular Clusters Collapsed Yet?" 2007, *Astrophysical Journal*, 656,L65.

- Eggleton, P.2006, Evolutionary Processes in Binary and Multiple Stars. Cambridge Astrophys. No.40. Cambridge, UK, Cambridge Univ Press.
- Elmegreen B.G., "Exponential Galaxy Disks from Stellar Scattering", 2007, Astrophysical Journal 668, 1064.
- Elmegreen, B.G. 2006. "Radial Profiles of Star Formation in the Far Outer Regions of Galaxy Disks" 2006, *Astrophysical Journal*, 648,572.
- Elmegreen, B. G., and Mathieu, R. D. 1983. "Quiescent formation of bound galactic clusters", 1983, MNRAS, 203, 305
- Faustini, F., Molinari, S., Testi, L. and Brand, J. 2009."Properties of stellar clusters around highmass young stars," 2009, Astronomy & Astrophysics, 503,801.
- Feller, W. 1980, "On the time distribution of so-called random events.", *Physics Review*, 57, 906
- Field, G.B. and Saslaw, W.C. 1965. "A Statistical Model of the Formation of Stars and Interstellar Clouds" 1965, *Astrophysical Journal*, 142, 568.
- Figer, D. F., McLean, I. S. and Morris, M. 1965. "The Pistol Star" 1999, *Astrophysical Journal*, 514,202.
- Figer, D.F, MacKenty, J.W, Robberto, M., Smith, K., Najarro, F. *et al.*,2006. "Discovery of Twin Wolf-Rayet Stars Powering Double Ring Nebulae" 2006, *Astrophysical Journal*,643,1166.
- He'non, M. 1969. "Asteroid 2012 XE133: a transient companion to Venus", 1969, Astronomy & Astrophysics, 2,151.
- Harayama, Y., Eisenhauer, F. and Martins, F. 2008. "The Initial Mass Function of the Massive Star-forming Region NGC 3603 from Near-Infrared Adaptive Optics Observations", 2008, Astrophysical Journal, 675,1319.
- Kanjilal, T. and Basu, B. 1997. "The evolution of star formation efficiency and mass spectrum during the formation of Star cluster",1997, *Indian Journal of Physics*, 121,131.
- Kruijssen, J.M.D. 2008."Explaining the mass-to-light ratios of globular clusters", 2008, *Astronomy & Astrophysics*, 486, L21.
- Kruijssen, J.M.D. and Mieske, S. 2009."Dissolution is the solution: on the reduced mass-to-light ratios of Galactic globular clusters", 2009, Astronomy & Astrophysics, 500,785.
- Larson, R. B. 1973."A Simple Probablisic Theory of Fragmentation", 1973, Monthly Notices of the Royal Astronomical Society, 161, 133.
- Mackey, A.D. and Gilmore, G.F. 2003."Surface brightness profiles and structural parameters for 53 rich stellar clusters in the Large Magellanic Cloud",
- 2003, Monthly Notices of the Royal Astronomical Society 379,151.

- Martins, F., Schaerer, D., and Hiller D.J. 2005."O stars with weak winds: the Galactic case" 2005, *Astronomy & Astrophysics*,436,1049.
- Maschberger, T. and Clark, C. J. 2008. "Maximum stellar mass versus cluster membership number revisited", 2008, Monthly Notices of the Royal Astronomical Society, 391,711.
- McLaughlin, D.E. and Van der Marel, R.P. 2005."A Population of Massive Globular Clusters in NGC 5128", 2005, *Astrophysical Journal*, 161,304.
- McCrady, N. and Graham, J.R. 2007."Super Star Cluster Velocity Dispersions and Virial Masses in the M82 Nuclear Starburst", 2007, *Astrophysical Journal*,663,844.
- Mengel. S.and Tacconi-Garman, L.E. 2003. "Medium resolution 2.3 m spectroscopy of the massive Galactic open clusterWesterlund 1", 2007, *Astronomy & Astrophysics*,466,151-55.
- Murray, S.D. and Lin, D.N.C. 1989a. "Coalesence, Star Formation and the Cluster Initial Mass Formation", 1989a, *Astrophysical Journal*, 339,933.
- Murray, S.D., and Lin, D.N.C. 1989b. "On the Coagulation and Size Distribution of Pressure Confined Cores" 1989b, *Astrophysical Journal*, 346,155.
- Nakano, T. 1966. "A Relativistic Field Theory of an Extended Particle, I", 1966, *Prog. Theory Phys*, 36,515.
- Parkar, R.J. and Goodwin, S.P. 2007. "Do O-stars form in isolation?" 2007, Monthly Notices of the Royal *Astronomical Society*,380,1271.

- Perina, S., Barmby, P., Beasley, M.A., Bellazzini, M. and Brodie, J.P. *et al.*, 2009."An HST/WFPC2 survey of bright young clusters in M31", 2009, *Astronomy & Astrophysics*,494,933.
- Pfalzner, S. 2009. "Formation and dissolution of leaky clusters", 2009, *Astronomy & Astrophysics*, 498,L37.
- Price, D.J. and Bate, M.R. 2009."Inefficient star formation: the combined effects of magnetic fields and radiative feedback", 2009, Monthly Notices of the Royal Astronomical Society, 398,33.
- Richer, H.B., Fahlman, G.G. and Buonanno, R. *et al.*, 1999."Are Blue Stragglers Mixed during Collisions?", 1991, *Astrophysical Journal*, 381, 147.
- Salpeter, E.E. 1955. "The Lumonisity Function and Stellar Evoolution", 1955, *Astrophysical Journal*, 121, 161.
- Silk, J. 1977."On the Fragmentation of Cosmic Gas Clouds. I. The Formation of Galaxies and the First Generation of Stars", 1977, *Astrophysical Journal*, 214,152.
- Spitzer L. 1987. In Dynamical Evolution of globular Clusters (Princeton, NJ, Princeton University Press), 1987,191.
- Wolf, M., and Vadimir, V. 1986. "The simulation of the initial mass function and star formation efficiency", 1986, Astrophysics Space Sci., 128, 229.
- Zhang, Q. and Fall, S. M. 1999."The Mass Function of Young Star Clusters in the "Antennae" Galaxies", 1999, Astrophysical Journal 527,L81.
- Zwart,S.F.P., McMillan,S.L.W. and Gieles, M. 2010. "Young Massive Star Clusters", 2010, Annual Review of Astronomy & Astrophysics,48,431.