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Application of Fuzzy Regression methodology for modeling of light penetration within wheat canopy

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ABSTRACT

Penetration of light within the crop canopy is essential for indicating growth and productivity of the crop. Radiation falling on crop is compartmentalized into absorption, reflection and transmission. These processes are regulated by the leaf area index (LAI) and plant height. It has been assumed that absorbed and reflected photosynthetic active radiation (APAR, RPAR) are the function of LAI and transmitted PAR (TPAR) is the function of LAI and plant-height. With these assumptions simple linear and multiple linear regression (MLR) models were developed to predict the estimated values of APAR, RPAR and TPAR, to predict the light penetration within wheat canopy. MLR and fuzzy regression (FR) methodology were applied to know the superiority of methodology for modeling of light penetration within canopy. In present investigation it was observed that FR is superior to MLR for modeling of light penetration within wheat canopy because average width of FR was less than that of MLR.

Keywords: FR, Penetration, Light, PAR, LAI, Plant height, Wheat

1. Introduction

The measurement of absorption, transmission and reflection of PAR are necessary to understand the compartmentation of radiation components within the crop canopy. Quantification of light interception within the crop canopy provides vital information on crop physiological process leading to proper explanation for yield variation (Gajjar *et al*., 1994; Jaybhaye *et al*., 2001; Wang *et al*., 2004; Basu *et al*., 2013. Radiation interception by the crop does not include the ground reflected part of the estimated absorption (Gallo and Daughtry, 1986). Absorption, transmission and reflection of PAR depend on the leaf area index and height of crop stand (Jena *et al*., 2010; Basu *et al*., 2013). If a statistical model for light penetration based on LAI and plant height can be developed, it would be immensely helpful for understanding the radiation compartmentation within the crop canopy. In this situation, number of radiation sensors may be reduced without compromising the accuracy in this estimation. A few workers have attempted this approach (Rosati *et al*., 2001; Sarlikioti *et al*., 2011) however no such attempt has been observed in Indian field crops.

Zadeh *et al*. (1965) describes the fuzzy uncertainty with ambiguity and vagueness and introduces the theory of fuzzy to build such a system as needed to deal with ambiguous and vague sentences or information. Tanaka *et al*. (1982) first proposed a study of fuzzy linear regression (FLR) model. Venus Marza and Mir Ali Seyyedi (2009) introduced a new Fuzzy Multiple Regression approach, which has the higher accuracy than other methods for estimating. In the present study we have tried to estimate the PAR penetration within wheat canopy using the biological characters *viz*. LAI and plant height using fuzzy regression (interval coefficient method) and multiple regression technique. We compared these two techniques for evaluating their accuracy for interpreting the result. The result may be encouraging to future works.

2. Material and methods

In this study, a model was developed based on a stand of wheat crop (variety PBW-343) adopted from an experiment conducted for three years (2009-12) in the winter season, at the BCKV, Research farm (22.56°N and 88.32°E), Kalyani, West-Bengal, India. The crop was sown on 25th November and 20th December in a plot of 6 X 5m having three replications in a split - plot design where the dates of sowing were the main plot treatment and three irrigation levels (I_1, I_2 and I_3) were the sub plot treatments. In the present study, the data at 11:30 h was considered starting from 30 days after emergence to the milking stage from week 1 to week 8 with the crop growth characters *i.e*. LAI and plant height because the sun remains at the zenith and maximum insolation is received, at 11:30 h.

Data was analyzed using LR, MLR and FR methodologies with the help of SAS 9.3 software. According to problem,

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$Y_1 = f(X_1)$ .... (1)
$Y_2 = f(X_1)$ .... (2)
→ Applied LR model from week 1 to week 8 for each two dates of sowing for three years

$Y_3 = f(X_1, X_2)$ .... (3) → Applied MLR model

where, Y_1 = APAR , Y_2 = RPAR , Y_3 = TPAR, X_1 = LAI, X_2 = Plant height

Using the above three equations or models, we predicted the estimated values for APAR, RPAR and TPAR to construct the pathway for light penetration within wheat canopy *i.e.* $Y\left(=\hat{Y_3}\right) = f\left(\hat{Y_1}, \hat{Y_2}, X_2\right)$........... (4)

We have applied two methodologies *viz*. MLR and FR for fitting the eqn. (4) to evaluate the superiority of methodology.

Multiple linear regression methodology

It is a very powerful technique and is extensively used in agricultural research. This technique estimates linear relationship between response (Y) and explanatory variables (X_i). The model is expressed as

$Y = b_0 + b_1X_1 + \ldots + b_nX_n + \varepsilon$ .... (5)

where, b'$_s$ are parameters and ε is the error term. The parameters are generally estimated using method of least squares. A good description of various aspects of multiple linear regression methodology is given in Draper and Smith (1998). One drawback of this methodology is that the underlying relationship is assumed to be crisp or precise but in the realistic situation, the relationship is not a crisp function instead contains vagueness or impreciseness. Due to the assumption of crisp relationship some important information may be lost therefore the technique of fuzzy regression has been developed to solve agricultural research problems.

Fuzzy regression methodology

Mc Cauley - Bell *et al*. (1999) and Sanchez and Gomez (2003a, b, 2004), used FR in their analysis. The former used it to predict the relationship of known risk factors to the onset of occupational injury. Their models took the general form corresponding to eqn. (5), following Tanaka *et al*. (1982):

$Y = A_0 + A_1X_1 + \ldots + A_nX_n$ .... (6)

where, Y= fuzzy output , A_i = fuzzy coefficients, i = 1,2,....,n, X_i = n-dimensional non-fuzzy input vector

Our aim is to estimate these parameters. It is assumed that A_i's symmetric fuzzy numbers (*i.e.* vagueness is expressible as equidistant from the center) and so can be represented by intervals. For example, A_i can be expressed as fuzzy set given by :

$A_0 = \langle a_{0c}, a_{0w} \rangle$ .... (7)

where, a_{0c} is centre and a_{0w} is radius or vagueness associated and the parameters are estimated by minimizing total vagueness in the model, *i.e.* sum of radii of predicted intervals. This can be visualized as the following linear programming problem (Tanaka 1987):

Minimize

$$\sum_{j=1}^{m}\left(a_{0w} + a_{1w}\left|x_{1j}\right| + \ldots + a_{nw}\left|x_{nj}\right|\right)$$

Subject to

$$\left\{\left(a_{0c} + \sum_{i=1}^{n} a_{ic}x_{ij}\right) - \left(a_{0\omega} + \sum_{i=1}^{n} a_{i\omega}x_{ij}\right)\right\} \le y_j$$

$$\left\{\left(a_{0c} + \sum_{i=1}^{n} a_{ic}x_{ij}\right) + \left(a_{0\omega} + \sum_{i=1}^{n} a_{i\omega}x_{ij}\right)\right\} \ge y_j \text{ and } a_{i\omega} \ge 0$$

To solve the above linear programming problem, Simplex procedure (Taha 1997) is generally employed.

3. Results and discussion

The MLR and FR methodologies have been applied for comparing the superiority of methodology for modeling of light penetration within wheat canopy. For that application, the results showed that Y_1 (APAR) and Y_2 (RPAR) were well fitted in the second date of sowing (20th December) with R^2 = 0.61 and R^2 = 0.52 in week 4 and week 7 by LR model but Y_3 (TPAR) was fitted well in the first date of sowing (25th November) as well as second date of sowing (20th December) with R^2 = 0.64 in week 3 and R^2 = 0.66 in week 4 by MLR model. According to these R^2 values of Y_1, Y_2 and Y_3 models, we predicted the estimated values *i.e.*, $\hat{Y_1}, \hat{Y_2}$, and $\hat{Y_3}$ but in case of Y_3 there were two R^2 values so we take R^2 = 0.66 $\left(R^2_{wk4} > R^2_{wk3}\right)$ and take its estimated values. After getting estimated values of APAR, RPAR and TPAR we predicted the model for light penetration within wheat canopy for week 4 only.

The relationship for modeling of light penetration within wheat canopy be –

$$Y\left(=\hat{Y_3}\right) = f\left(\hat{Y_1}, \hat{Y_2}, \hat{Y_2}\right) \quad (8)$$

Table 3: Fitting of MLR and FR

Actual TPAR	Predicted TPAR (MLR model)			Predicted TPAR (FR model)		
	Upper	Lower	width	Upper	Lower	Width
6.92	7.37	6.87	0.51	6.91	6.9	0.02
9.27	9.72	9.22	0.51	9.26	9.24	0.02
7.76	8.22	7.71	0.51	7.76	7.74	0.02
9.01	9.46	8.96	0.5	9.01	8.99	0.02
12.25	12.72	12.21	0.51	12.25	12.23	0.02
5.97	6.44	5.91	0.52	5.96	5.94	0.02
15.87	16.34	15.83	0.51	15.86	15.85	0.02
16.49	16.99	16.46	0.53	16.49	16.47	0.02
11.28	11.77	11.24	0.53	11.27	11.26	0.02
32.19	32.66	32.18	0.48	32.19	32.18	0.01
17.37	17.9	17.35	0.54	17.37	17.36	0.02
16.24	16.76	16.22	0.54	16.24	16.23	0.02
33.52	34	33.5	0.49	33.51	33.5	0.01
18.09	18.64	18.07	0.56	18.09	18.07	0.01
15.18	15.73	15.16	0.56	15.18	15.17	0.02
29.03	29.55	29.02	0.53	29.03	29.02	0.01
18.14	18.7	18.13	0.57	18.14	18.13	0.01
8.11	8.68	8.09	0.59	8.11	8.09	0.02
29.69	30.12	29.66	0.45	29.69	29.68	0.01
39.64	40.08	39.63	0.46	39.64	39.63	0.01
28.02	28.46	27.99	0.47	28.01	28	0.01
45.40	45.82	45.39	0.43	45.4	45.39	0.01
44.73	45.17	44.72	0.46	44.72	44.71	0.01
37.60	38.05	37.59	0.46	37.6	37.59	0.01
53.28	53.72	53.28	0.44	53.28	53.27	0.01
55.42	55.88	55.43	0.45	55.42	55.41	0.01
44.31	44.78	44.31	0.47	44.31	44.3	0.01
Average width			0.50	**Average width**		**0.01**

Table 1: MLR SAS Output

Variable	DF	Parameter Estimate	Standard Error	t Value	$Pr > \lvert t \rvert$
Intercept (predicted TPAR) [Y(= $\hat{Y}_3$)]	1	130.60	0.019	8466.27	<.0001
$\hat{Y}_1$ (predicted APAR)	1	-1.07	0.001	-13072	<.0001
$\hat{Y}_2$ (predicted RPAR)	1	0.02	0.007	1.51	0.1435
X_2 (plant height)	1	- 0.64	0.003	-4001.2	<.0001

From table 1, the fitted model for MLR is

$$Y (= \hat{Y}_3) = 130.60 - 1.07 * \hat{Y}_1 + 0.02* \hat{Y}_2 - 0.64 * X_2 \quad (9)$$

Table 2: FR SAS Output

ac	aw
130.5766298	0.000000000
-1.0698197	0.000082301
0.0083922	0.000000000
-0.6438710	0.000028956

From table 2, the fitted model for FR is

$$Y (= \hat{Y}_3) = <130.58, 0> + <-1.07, 0.00008> * \hat{Y}_1 + <0.00839, 0> * \hat{Y}_2 + <-0.64387, 0.00003> * X_2$$

In order to compare performance of above two approaches, *viz*. MLR and FR methodology, width of prediction intervals corresponding to each observed value of response variable computed. For the former, upper and lower limits of prediction interval are computed from the prediction eqn. (9) by taking the coefficient as their corresponding estimated values plus standard error, (table1) *i.e.*

$$Y (= \hat{Y}_3) = (130.58+ 0.019)+(-1.07 + 0.001)* \hat{Y}_1 + (0.02 + 0.007)* \hat{Y}_2 + (-0.64+ 0.003)* X_2$$ and

$$Y (= \hat{Y}_3) = (130.58- 0.019) + (-1.07 - 0.001)* \hat{Y}_1 + (0.02 - 0.007)* \hat{Y}_2 + (- 0.64-0.003)* X_2$$

Further for fuzzy regression model, the prediction eqn. for computing upper and lower limit, obtained from eqn. (10) are respectively (table2),

$$Y(= \hat{Y}_3)=<130.58+0>+<-1.07 + 0.00008 > \hat{Y}_1 *+ < 0.00839+0>* \hat{Y}_2 + <-0.64387+0.00003>* X_2$$

and

$$Y(= \hat{Y}_3)=<130.58-0>+<-1.07-0.00008 > * \hat{Y}_1 + <0.00839-0> * \hat{Y}_2 + <-0.64387-0.00003>* X_2$$

Average width for MLR model was found to be 0.50, while that for FLR model was only 0.01 (Table 3), indicating thereby the superiority of fuzzy regression methodology. Similar kind of findings was reported by (Kandala and Prajneshu, 2003) who demonstrated the applicability of FLR methodology when the two explanatory variables (*viz*. plant height and leaf area index) and response variable (dry matter accumulation) are all crisp but underlying phenomenon is assumed to be fuzzy in nature. It was shown that widths of prediction intervals in respect of FLR model were much less than those for MLR model.

4. Conclusions

The predicted interval computed using FLR methodology have much shorter average width as compared to that obtained using least square method. This implies that FLR procedure is more efficient than MLR. The main message emerging out of this study is that correct methodology to determine the modeling of light penetration within wheat canopy of FLR rather than ordinary least squares.

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