RASHI 3 (2) : 01-09 (2018) ISSN : 2581 9755

Different aspects of Reliability

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Received : 09-03-2019 ; Revised : 24-03-2019 ; Accepted : 31-03-2019

ABSTRACT

Although Reliability started during second World War, its mathematical development started during 1960s. Starting from the genesis, here we explore different kinds of systems having practical applications viz. coherent system, in general, and series, parallel and k-out-of-n systems, in particular. We shall also discuss different properties of path sets and cut sets and their use in finding reliability of complex systems. We also discuss in brief different kinds of redundancies to enhance the reliability of a given system.

***Keywords* :** Ageing classes, censoring, coherent system, redundancy, structure function

1. Genesis

Theory of Reliability has its roots during World War II when Abraham Wald, as a member of the Statistical Research Group, applied his knowledge of statistics to different problems faced during world war. To be more specific, he used his knowledge of sampling inspection techniques and method of sequential analysis to study the damage caused to the aircrafts and minimize losses of the aircrafts from enemy fire. Important advances in the theory of reliability were made during 1950s. One important work in this direction is due to Epstein and Sobel (1953) where estimates are obtained for the mean lifetime of the systems based on Type-II censored data drawn from an exponential distribution. It is to be mentioned here that the data are said to be censored if units are removed from consideration prior to their failure and the test is completed prior to all units' failure. It is to be noted here that the units may be removed, for example, when they fail because of other failure modes than the one being measured. There are different kinds of censoring and their generalizations. However we want to discuss here very basic censoring schemes. If all working units have the same lifetime, and the test is concluded before all units have failed, it is called singly censored. When units are removed at various times the scheme is called multiply censored scheme (See Fig. 1 for diagrammatic representation). A censoring scheme is called left censoring if the failure times for some units are known to occur only before some specified time whereas it is called right censoring if the failure times for some units are known only to be after some specified time. Let us take an example to make it more clear. Suppose a person infected with HIV comes to a doctor and gets diagnosed to have AIDS. The time period between being

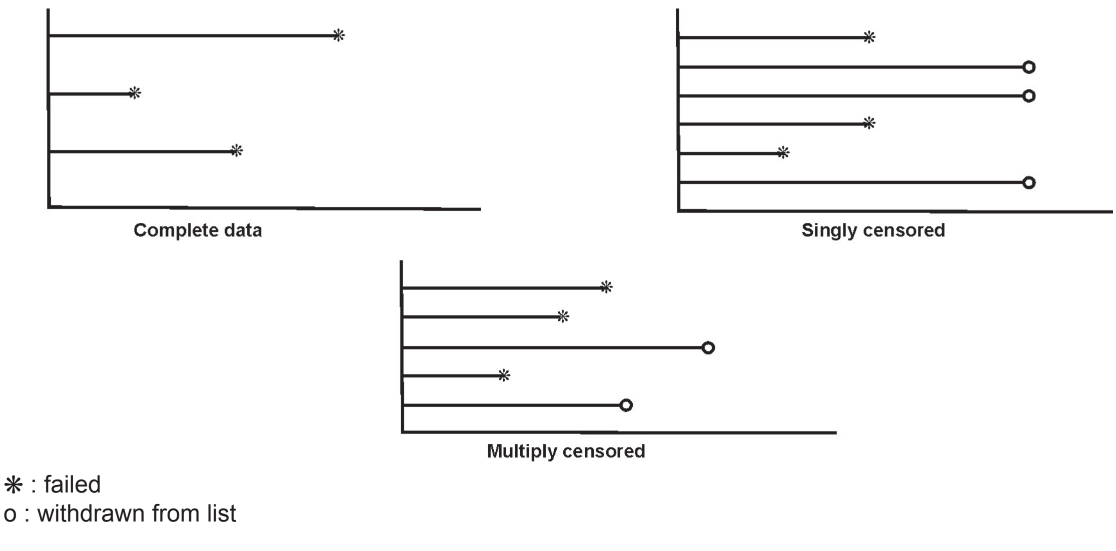

Fig. 1 : Different types of censoring

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diagnosed as AIDS patients and getting infected with HIV is generally called incubation period, which is not known. When a person is diagnosed as having AIDS, the treatment starts there (we call it time zero). At this point we have only information that the person got infected with HIV sometime before time zero (and no more information we have). Data on these patients are called left-censored data. When a person getting treated regularly in a doctor's chamber, stops visiting the doctor, then we have the only information that the person was alive when he visited the doctor last, and no information about him after that. The data on such patients are considered as right-censored data.

In order to study some systems, say electric bulbs, we may put n (a pre-fixed number) bulbs into test and wait until all the bulbs fail in order to get their lives. However, this may not be feasible because the last bulb may fail after thousands of hours. In order to avoid this difficulty, we may fix some time (say t_0) after which the test will be terminated. We shall have complete information on the lives of the bulbs which will fail before time t_0 and the lives of the rest of the bulbs are censored. This kind of censoring is called Type-I censoring or time censoring. Note that, in this kind of censoring, the number of bulbs failed by time t_0 is a random variable. We may also decide to stop the experiment after r (a fixed number) bulbs fail. In this case, the time of running the experiment will be a random variable. This kind of censoring is called Type-II censoring.

James Esary, Albert Marshall, Frank Proschan and Sam Saunders, the members of the Core Team of Boeing Aircraft Company (a multinational corporation, founded byWilliam E. Boeing, that manufactures airplanes, rockets, missiles etc.), did a good amount of work on reliability. However, although Richard Barlow, Z.W. Birnbaum and Ingram Olkin were not in the core team, they contributed a lot in the development of reliability theory. They published their work on the three primary sub-fields of Reliability, *viz*. (*i*) *Structural Reliability* where different kinds of design of systems and how these designs influence the system lives are studied, (*ii*) *Stochastic Reliability* where we concentrate on modeling the lifetime characteristics of systems and their performance, and (*iii*) *Statistical Reliability* where we concentrate on the process of drawing inferences about general characteristics of systems from experimental data on their performance. Couple of best works in this direction are due to Birnbaum et al. (1961, 1966), Proschan (1963) and Barlow and Proschan (1965).

2. Some basic concepts

We start this section by giving the definition of reliability of a system.

Definition 2.1 *Reliability is the* **probability** *that a system will perform its intended function* **satisfactorily** *for a* **specified period of time** *under a* **given set of conditions.**

In the above definition some terms are written in bold. These are some important points to remember while we talk of reliability of a system. If X, having distribution function F and survival function $R \equiv 1-F$, denotes the lifetime of a system, then the reliability of the system at time t is defined as the probability that the system has survived at least for t units of time, *i.e.*, reliability at time t, denoted by $R(t)$, is defined as $R(t) = P(X > t)$. In order to compare performance of two systems we must prefix the required satisfaction level. Otherwise, the reliability comparison will not be fair. Finally, before such comparison is made, two systems should be kept under same prior conditions in order to have fair comparison.

Let N identical units (may be living organism or a system of components) be put to test at time 0 and let $N\_s(t)$ be the number of surviving items at time t. Then $N - N\_s(t) = N_f(t)$ is the number of failed items by time t. Here, by failed item we mean that the item may not actually fail, but the performance has gone below the prefixed satisfaction level. The estimate of reliability $R(t)$ is given by $\hat{R}(t) = \frac{N_s(t)}{N}$ and that of the distribution function is $\hat{F}(t) = \frac{N\_f(t)}{N}$.

Let f be the density function corresponding to F. Then the probability that an item which is known to survive at least for t units of time will fail within the interval $(t, t+\delta t]$, for very small value of δt, is given by

$$P(t < X \leqslant t + \delta t | X > t) \approx \frac{f(t)\delta t}{R(t)} = \lambda(t).\delta t,$$

where $\lambda(t) = \frac{f(t)}{R(t)}$ is known as the (instantaneous) failure rate (FR), which is sometimes called hazard rate. This can be estimated by

$$\hat{\lambda}(t) \approx \frac{1}{N_s(t)} . \frac{N_s(t) - N_s(t+\delta t)}{\delta t}.$$

Let the items which are put to test will fail one at a time, and let the failure times be denoted by t_0 (= 0), $t_1, t_2, \ldots, t_N$, where the i^{th} item has failed at time t_i, $i = 1, 2, \ldots, N$. Then we get the mean time to failure (MTTF) as

$$\begin{aligned} MTTF &= \int_0^\infty R(t)\, dt \\ &\approx \int_0^\infty \frac{N_s(t)}{N}\, dt \\ &= \sum_{i=1}^{N} \int_{t_{i-1}}^{t_i} \frac{N_s(t)}{N}\, dt \\ &= \frac{1}{N}\left[Nt_1 + (N-1)(t_2 - t_1) + \ldots + (N-(N-1))(t_N - t_{N-1})\right] \\ &= \frac{1}{N}\sum_{i=1}^{N} t_i. \end{aligned}$$

This is pictorially described in Figure 2.

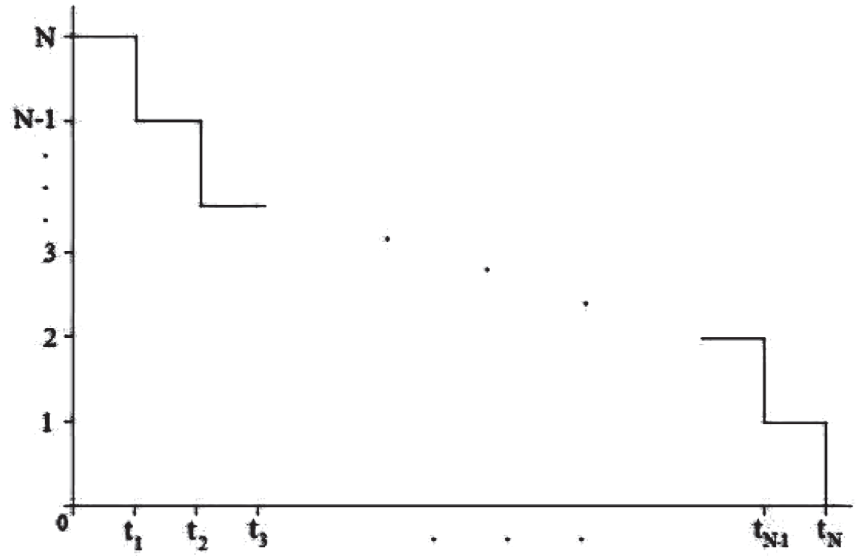


Fig. 2 : Mean time to failure

3. Ageing classes

Ageing, an inherent property of a unit, is an important phenomenon in reliability theory. By ageing, we mean a mathematical specification of degradation of item over time. It is a phenomenon which tells that an older system, in some statistical sense, has a shorter remaining lifetime than a newer one. When we talk of ageing we generally mean positive ageing whereas by negative ageing (sometimes called antiageing) we mean mathematical specification of upgradation of an item over time. Clearly, negative ageing is a beneficial ageing. Some very common ageing classes are described below.

Definition 3.1 *A distribution is said to have (be)*

(*i*) *increasing failure rate* (*IFR*) *if* $\lambda(t)$ *is increasing in t;*

(*ii*) *decreasing failure rate (DFR) if* $\lambda(t)$ *is decreasing in t;*

(*iii*) *constant failure rate (CFR) if* $\lambda(t)$ *is constant;*

(*iv*) *linear failure rate (LFR) if* $\lambda(t) = a + bt$, where a and b are constants.

Note that $\lambda(t) = \lambda$, a constant, if and only if MTTF $= \frac{1}{\lambda}$. Exponential distribution is characterized by constant failure rate. One may wonder whether it is possible in practice to have any system whose lifetime distribution is exponential. To answer this, let us consider the life of human being. At the time of birth the failure rate (of the life of a new-born baby) is very high which comes down, in a very short period of time (say t_1), to λ (a constant) and generally remains more or less constant, and then start to increase (say at time t_2) in the old age. This kind of failure rate is known as bathtub-shaped failure rate. We may formally define this as under.

Definition 3.2 *A distribution is said to have bathtub (BT) shaped failure rate if there exist* $0 \leq t_1 \leq t_2$ *such that* $\lambda(t)$ *(strictly) decreases in* $0 \leq t \leq t_1$*, it is constant in* $t_1 \leq t \leq t_2$ *and (strictly) increases in* $t \geq t_2$*. Thus, for a distribution having BT shaped failure rate,* $\lambda(t)$ *will be of the form*

$$\lambda(t) = \begin{cases} \lambda_1(t), & 0 \leqslant t \leqslant t_1 \\ \lambda, & t_1 < t < t_2 \\ \lambda_2(t), & t \geqslant t_2 \end{cases}$$

where $\lambda_1(t)$ *decreases in* $[0, t_1]$ *and increases in* $[t_2, \infty)$.

It is to be mentioned here that, as a particular case of BT shaped distribution, we get an IFR distribution when $t_2 = 0$, a DFR distribution for $t_1 \to \infty$, and exponential distribution if $t_1 = 0$ and $t_2 \to \infty$. It is to be mentioned here that BT distributions are neither closed under convolution nor closed under mixture nor closed under formation of coherent system[1].

However, if X has a BT shaped failure rate then, for any increasing function ϕ, $\phi(X)$ also has BT shaped failure rate. Note that, in the BT shaped distribution, the lifetime during (t_1, t_2) can be considered to have more or less constant failure rate (giving the distribution as exponential). Below we give some more cases where exponential distribution can arise.

(a) If FRs are determined by completely random independent events not associated with the age of the system, an exponential distribution will result.

(b) Let a system be comprised of many components acting independently and let an individual component failure causes system failure. In a renewal process where a failed component is immediately replaced by a new one, the system, after sometime, will reach steady state, *i.e.*, after sometime, a constant number of failures per unit time will be observed.

(c) Let independent loads be applied on components of a (series) system (of n components) at a fixed interval of time (δt), and let the probability of failure of a component due to the loads be p, a small constant. Then the component reliability is given by $R = 1 - p$, and the system reliability at time t, $R_n(t)$, can be computed as

$$\begin{aligned} R_n(t) &= (1-p)^n \\ &= e^{n \ln(1-p)} \\ &\approx e^{-np} \quad (\text{since } \ln(1-p) \approx -p) \\ &= e^{-(p/\delta t)t}, \ [n = t/\delta t] \\ &= e^{-\lambda t}, \ [\lambda = p/\delta t = \text{constant}] \end{aligned}$$

which is the survival function of an exponential distribution with failure rate λ.

A linear failure rate distribution having failure rate $\lambda(t) = a + bt$, $a \geq 0$, $b > 0$ is denoted by $LFR(a, b)$, which is IFR. This distribution has reliability

$$R(t) = e^{-(at+bt^2/2)}$$

and density function is given by

$$f(t) = (a+bt)e^{-(at+bt^2/2)}, \ t > 0.$$

[1] To be formally defined later

It is to be noted that Rayleigh distribution, a very important distribution in reliability, is obtained as a particular case of linear failure rate distribution (obtained when the constant a is zero). A LFR distribution is a skewed unimodal distribution on $[0,\infty)$, and is closed under formation of series system (to be discussed later). It is to be noted that if $X \sim LFR(a, b)$ then the residual random variable, $X_t = (X - t \mid X > t)$, has the $LFR(a + 2bt, b)$ distribution.

4. Some special systems

Let $X_1, X_2, \ldots, X_n$ be the independent component lives of the system with X_i having reliability $R_i(t)$ at time t. Define, for $i = 1, 2, \ldots, n$,

$$X_i = \begin{cases} 1 & \text{if the } i^{th} \text{ component is working} \\ 0, & \text{otherwise.} \end{cases}$$

Let X be the lifetime of the system formed out of the components having lifetimes $X_1, X_2, \ldots, X_n$. Define $\phi : \{0, 1\}^n \rightarrow \{0, 1\}$ as

$$\phi(x_1, x_2, \ldots, x_n) = \begin{cases} 1, & \text{if the system is working} \\ 0, & \text{otherwise.} \end{cases}$$

Here ϕ is called the structure function of the system. A system is said to be a coherent system if each of the components is relevant (*i.e.*, the system does not have any component which has no role to play with the working of the system), and its structure function is increasing (in each argument keeping the others fixed).

The simplest coherent systems are the series and the parallel systems. A system is called a series system if it fails whenever any one of the components fails. A system is called parallel system if it works as long as at least one of the components works. Let a series system (having lifetime X) be formed out of n components having tifetimes $X_1, X_2, \ldots, X_n$. If the reliability of the i[th] component at time t is $R_i(t) = P(X_i > t)$, then the reliability of the system at time t is calculated as

$$\begin{aligned} R(t) &= P(X > t) \\ &= P[\min\{X_1, X_2, \ldots, X_n\} > t] \\ &= \prod_{i=1}^{n} R_i(t). \end{aligned}$$

If the component lives are *iid* (independent and identically distributed), with common reliability (at time t) of the components $p = R_1(t)$, then the reliability of the system at time t is given by $R(t) = p^n$. The reliability of the parallel system constructed out of these X_is is given by

$$\begin{aligned} R(t) &= 1 - P[\max\{X_1, X_2, \ldots, X_n\} \leqslant t] \\ &= 1 - \prod_{i=1}^{n}(1 - R_i(t)), \end{aligned}$$

which becomes $1 - (1 - p)^n$ if the component lives are iid with reliability p. Both the series and the parallel systems are generalized to the k-out-of n system which is again a particular case of coherent system. A k-out-of-n : G System (mostly known as k-out-of-n system) is a system which works as long as at least k of the n components work. The reliability at time t of a k-out-of-n system constructed out of *iid* components having lifetimes $X_1, X_2, \ldots, X_n$ is calculated as

$$\begin{aligned} R(t) &= P(\text{At least } k \text{ of } X_1, X_2, \ldots, X_n > t) \\ &= \sum_{i=k}^{n} P(\text{exactly } i \text{ of } X_1, X_2, \ldots, X_n > t) \\ &= \sum_{i=k}^{n} \binom{n}{i} p^i (1 - p)^{n-i}. \end{aligned} \tag{4.1}$$

Clearly, a series system is an *n*-out-of-*n* system whereas a parallel system is a 1-out-of-*n* system. From (4.1) we note that *R*(*t*) is a polynomial in *p*. This is known as reliability polynomial. Sometimes we also define *k*-out-of-*n* : *F* system. It is a system which fails with the failure of the k[th] component, *i.e.*, the system works as long as the number of failures is less than *k, i.e.*, number of working components is more than $n - k$, *i.e.*, the number of working components is at least $n - k + 1$. Thus, *k*-out-of-*n* : F system is same as $(n - k + 1)$-out-of-*n* : *G* system. Below we give some examples of *k*-out-of-*n* systems (cf. Balagurusamy, 1984).

(a) In an automobile with six cylinders it may be possible to drive the vehicle with four cylinders firing.

(b) A communication channel having three transmitters the message may be properly transmitted when at least two transmitters properly work.

(c) An aircraft having four engines flies whenever at least two engines are in proper working conditions.

(d) A bridge supported by 10 cables may require only 6 cables to perform the maximum load.

It is not difficult to verify that the structure function of a series system is given by

$\phi(\mathbf{x}) = \prod_{i=1}^{n} x_i$, that of parallel system is given by $\phi(\mathbf{x}) = 1 - \prod_{i=1}^{n}(1 - x_i)$ whereas, for a *k*-out-of-*n* system, it is

$$\phi(\mathbf{x}) = \begin{cases} 1, \text{if } \sum_{i=1}^{n} x_i \geqslant k \\ 0, \text{ otherwise.} \end{cases}$$

It is to be mentioned here that coherent system drastically restricts the number of possible functions mapping from $\{0, 1\}^n$ to $\{0, 1\}$. To be more specific, out of the 256 functions mapping from $\{0, 1\}^3$ to $\{0, 1\}$, only 5 correspond to coherent systems. However, it is surprising to note that in spite of this drastic reduction, the coherent systems of order n grows rapidly with n. For example, there are only 2 coherent systems of order 2, 5 coherent systems of order 3, 20 coherent systems of order 4, and more than a billion coherent systems of order 30. For more discussion on this, one may refer to Samaniego (2007). In Table 1, we give structure functions of coherent systems of different orders.

5. Path set and Cut set

A set of components *P* is said to be a path set if the system works whenever all the components in *P* work. A set, no proper subset of which is a path set, is said to be a minimal path set (MPS). A set of components *C* is said to be a cut set if the system fails whenever all the components in *C* fail. A cut set is a minimal cut set (MCS) if it has no proper subset that is also a cut set.

Table 1 : Structure functions of coherent systems

Order of Coherent System	Structure Function
Order-1	$\phi(x) = x_1$
Order-2	$\phi(\mathbf{x}) = x_1x_2 = \prod_i x_i$ (Series System) MPS: {1,2} MCSs: {1},{2} $\phi(\mathbf{x}) = 1 - (1 - x_1)(1 - x_2)$ (Parallel System) MCS: {1,2} MPSs: {1},{2}
Order-3	$\phi(\mathbf{x}) = x_1x_2x_3 = \prod_i x_i$ (Series System) MPS: {1,2,3} MCSs: {1},{2},{3} $\phi(\mathbf{x}) = 1 - (1 - x_1)(1 - x_2)(1 - x_3)$ (Parallel System) MCS: {1,2,3} MPSs: {1},{2},{3} $\phi(\mathbf{x}) = x_1[1 - (1 - x_2)(1 - x_3)]$ MPSs: {1,2},{1,3} MCSs: {1}, {2,3} $\phi(\mathbf{x}) = 1 - (1 - x_1)(1 - x_2x_3)$ MCSs: {1,2},{1,3} MPSs: {1}, {2,3} $\phi(\mathbf{x}) = 1 - (1 - x_1x_2)(1 - x_2x_3)(1 - x_3x_1)$(2-out-of-3 system) MPSs: {1,2}, {1,3}, {2,3} MCSs: {1,2}, {1,3}, {2,3}

5.1 Properties of MPS and MCS

The following properties of MPS and MCS are important to be noted because these will be used to find out reliability of complex systems.

(*i*) No MPS is a proper subset of any other.

(*ii*) The union of all MPSs is the set of all the components of the system.

(*iii*) No MCS is a proper subset of any other.

(*iv*) The union of all MCSs is the set of all the components of the system.

Note that it is possible to characterize all coherent systems of a given order n by these two properties of its MPS (MCS). Since by (*ii*), every component from 1 to n is a member of at least one MPS, the relevance of every component is guaranteed. If component k is not working and the system is also not working, then the system structure function ϕ will either remain equal to 0 or will increase to unity when component k is replaced by a working component.

Below we give structure function of different coherent systems. Computation of structure function for an arbitrary coherent system can be algebraically cumbersome. However, there is a connection between the structure function of a coherent system and its MPSs and MCSs which can be used to compute the structure function of a coherent system on using the MPSs and MCSs. Note that

(i) a system works if and only if all the components of at least one MPS work;

(ii) a system works if and only if at least one of the components in every MCS work.

Let P_j be the MPS of a system, $j = 1, 2, \ldots$, r. Define

$$p_j(\mathbf{x}) = \prod_{i\in P_j} x_i.$$

Then

$$\begin{aligned}\phi(\mathbf{x}) &= 1-\prod_{j=1}^{r}(1-p_j(\mathbf{x}))\\ &= \max_{1\leqslant j\leqslant r} p_j(\mathbf{x})\\ &= \max_{1\leqslant j\leqslant r}\ \min_{i\in P_j} x_i.\end{aligned}$$

Further, let C_j be the MCS of a system, $j = 1, 2, \ldots, k$. Define

$$c_j(\mathbf{x}) = 1-\prod_{i\in C_j}(1-x_i).$$

Then

$$\begin{aligned}\phi(\mathbf{x}) &= \prod_{j=1}^{k} c_j(\mathbf{x})\\ &= \min_{1\leqslant j\leqslant k} c_j(\mathbf{x})\\ &= \min_{1\leqslant j\leqslant k}\ \max_{i\in C_j} x_i.\end{aligned}$$

Let us consider the bridge system given in fig. 3. Since {1,4}, {1,3,5}, {2,5}, {2,3,4} are the MPSs and {1,2}, {4,5}, {1,3,5}, {2,3,4} are the MCSs for the bridge system, we have

$$\begin{aligned}\phi(\mathbf{x}) &= 1-[(1-x_1x_4)(1-x_1x_3x_5)(1-x_2x_5)(1-x_2x_3x_4)] \text{ (in terms of MPSs)}\\ &= [1-(1-x_1)(1-x_2)][1-(1-x_4)(1-x_5)][1-(1-x_1)(1-x_3)(1-x_5)].\\ &\quad [1-(1-x_2)(1-x_3)(1-x_4)] \text{ (in terms of MCSs)}.\end{aligned}$$

From the MPSs and MCSs we construct structures which are equivalent to the bridge structure, as given in fig. 4. The following important points may be noted.

(i) A coherent system is a parallel system in which each element is a series system in the components in the MPS.

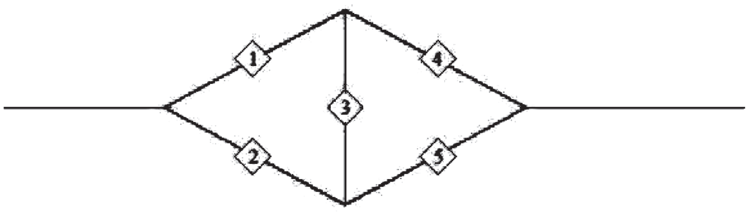


Fig. 3 : Bridge system

(ii) A coherent system is a series system in which each element is a parallel system in the components in the MCS.

(iii) For a system i with structure function ϕ_i, $i = 1, 2$, system 2 is better than system 1 if

$$\phi_1(\mathbf{x}) \leqslant \phi_2(\mathbf{x}), \text{ for all } \mathbf{x} \in \{0,1\}^n.$$

(iv) Since no system is better than a parallel system and no system is worse than a series system, we have

$$\prod_{i=1}^{n} x_i \leqslant \phi(\mathbf{x}) \leqslant 1 - \prod_{i=1}^{n}(1 - x_i).$$

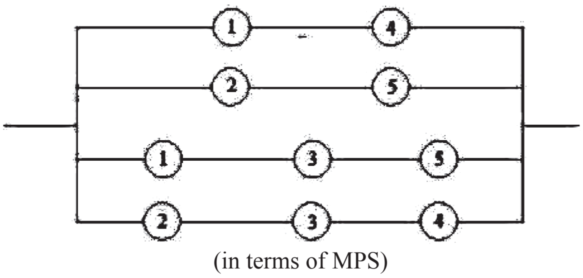
For a coherent system A we define its dual as another coherent system B if the MPSs of one are the MCSs of the other. Thus, $\phi^A(x) = 1 - \phi^B(1-x)$. From table 1, we see that a series system is dual of a parallel system of same number of components whereas a 2-out-of-3 system is dual of itself.

6. Component redundancy vs. system redundancy

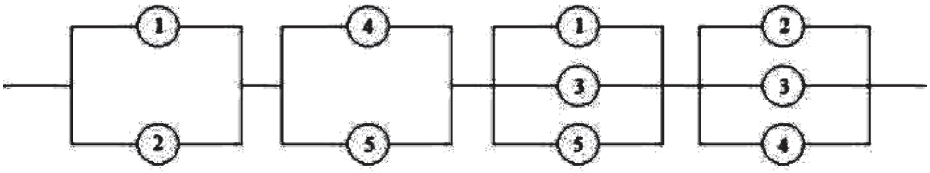
Suppose we have an n-component system with component lives $X_1, X_2, \ldots, X_n$ and m additional components (known as redundant components) having lifetimes $Y_1, Y_2, \ldots, Y_m$ $(m \geq n)$ are available to enhance the performance of the system by incorporating redundancy in it. Let the reliability of a system be enhanced in either of the following two different ways.

(*i*) Each component of the system may have one or more parallel components. This kind of redundancy is called Component Redundancy.

(*ii*) The entire system may be placed in parallel with one or more identical systems. This kind of redundancy is called System Redundancy.



(in terms of MPS)



(in terms of MCS)

Fig. 4 : Equivalent bridge structures

Let us take $m = n$. Further, let x_i be the realization of X_i and y_i be that of Y_i. Note that, if each x_i and y_i takes values from {0,1}, then we have $1 - (1 - x_i)(1 - y_i) \geq \min\{x_i, y_i\}$, which gives

$$\begin{aligned} \phi[1-(1-x_1)(1-y_1),\ldots,1-(1-x_n)(1-y_n)] &\geqslant \max\{\phi(\mathbf{x}),\phi(\mathbf{y})\} \\ &= 1-(1-\phi(\mathbf{x}))(1-\phi(\mathbf{y})), \end{aligned}$$

where $x = (x_1, x_2, \ldots, x_n)$ and $y = (y_1, y_1, \ldots, y_n)$. This means that component redundancy is superior to system redundancy. In order to compute the reliability of a coherent system, write $p_i = P(X_i = 1)$ and $p = (p_1, p_2, \ldots, p_n)$. Then the system reliability is given by

$$h(\mathbf{p}) = P(\phi(\mathbf{x}) = 1) = E[\phi(\mathbf{x})],$$

which is a multilinear function (*i.e.*, linear in every p_i). Thus, by replacing x_i by p_i in the structure function we get the survival function. By adopting this technique, the survival function of the bridge system (as discussed earlier) constructed out of iid components, is obtained as

$$h(p) = 2p^2 + 2p^3 - 5p^4 + 2p^5.$$

There are other kinds of redundancy as well. When the original component and the redundant component work together so that the life of the system is the maximum of the original component life and the redundant component life, the redundancy is called Active Redundancy (or Hot Standby), whereas if the redundant component starts working once the original component fails so that the system life is the convolution of the original component life and the redundant component life, the redundancy is called Standby Redundancy (or Cold Standby). Sometimes, the cold standby takes time to start working once the original component fails. The time between failure of the original component and the time when the redundant component starts working is known as lead time. Sometimes we cannot allow any positive lead time. For example, in case of shadowless lamp used in case of surgery, the 'censoring and switching device' is not allowed to take any positive lead time. In this case the redundant components are used neither in the active state nor in the cold state. Here the redundant component is allowed to work in a state where its failure rate is non-zero but less that the failure rate of its active state. This kind of redundancy is called warm redundancy or warm standby. There is a vast literature on redundancy. However, some important works in this direction are due to Gordon (1957), Singh and Singh (1997), Misra et al. (2009), Hazra and Nanda (2014, 2015), Zhao *et al*. (2015) to mention a few.

7. Conclusion

In this note a brief overview of reliability theory covering different aspects of it has been given so that those who are new in the field of reliability can get some idea to start working on this field.

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